Derivation of Hardy-Weinberg Equilibrium

Generation 1:

Genotype frequencies

Frequency MM = u

Frequency MN = v

Frequency NN = w

Remember, frequencies sum to 1, so u + v + w = 1

Allele frequencies

Frequency M = u + v/2 = p

Frequency N = w + v/2 = q = 1-p

Remember, frequencies sum to 1, so p + q = 1 if there are only two alleles.

Mating Type	**MM**	**MN**	**NN**	**frequency of mating**
MM x MM	1	0	0	u * u
MM x MN	0.5	0.5	0	2uv
MM x NN	0	1	0	2uw
MN x MN	.25	.5	.25	v * v
MN x NN	0	.5	.5	2vw
NN x NN	0	0	1	w * w

Generation 2 (and all subsequent generations, provided assumptions are met)

Genotype frequencies

Frequency MM = $1* u^2 + 0.5*(2uv) + 0.25*v^2$
= $u^2 + uv + 0.25v^2$
= $(u + v/2)^2$
= p^2

Frequency MN = $0.5*(2uv) + 1*(2uw) + 0.5*v^2 + 0.5*(2vw)$
= $uv + 2uw + 0.5v^2 + vw$
= $2(u + v/2)(w + v/2)$
= $2pq$

Frequency NN = $0.25*v^2 + 0.5*(2vw) + 1*w^2$
= $0.25v^2 + vw + w^2$
= $(v/2 + w)^2$
= q^2

Allele frequencies

Frequency M = $p^2 + 0.5 * 2pq$
= $p^2 + pq$
= $p*(p + q)$
= p

Frequency N = $q^2 + 0.5 * 2pq$
= $q^2 + pq$
= $q*(q + p)$
= q